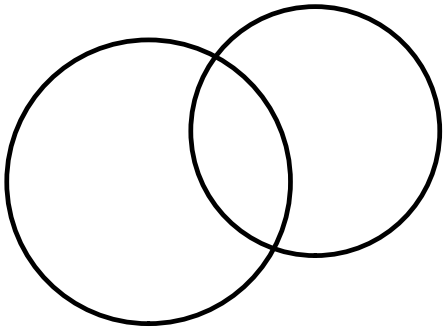
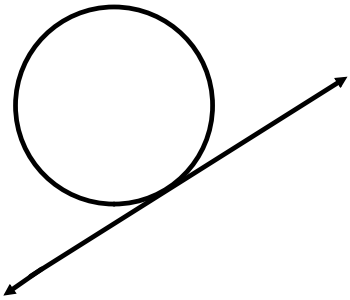
Circles: points of intersection and tangency

How would you find out where two circles intersect? How about where a circle is tangent to a line?





Where any objects intersect or touch, the equations for those objects must be equal.

That means we can use algebraic techniques to find the points where the two objects intersect or touch.

For example: substitution and setting two equations equal to each other.

Find where the line intersects the circle.

$x^2 + y^2 = 65$
$y = x - 3$

At any point where the line would intersect the circle the two equations would be equal. That is, they would both have the same solution at those points.

In this example we can use substitution to find those points. (Why can we use substition?)

$x^2+y^2 = 65$

Since $y = x-3$ we can substitute $x-3$ for y in this equation.

$x^2 + (x-3)^2 = 65$	substitute x - 3 for y
$x^2 + x^2 -3x -3x +9 = 65$	expand $(x-3)^2$
$2x^2 - 6x - 56 = 0$	group like items
$x^2 - 3x - 28 = 0$	divide by 2 to simplify
$(x - 7)(x+4) = 0$	factor (important!)
$x - 7 = 0$; $x = 7$, $y = 4$	solve for x, then y
$x + 4 = 0$; $x = -4$, $y = 1$	

The line intersects the circle at (7,4) and (-4,1)

Factoring - review

Recall that we can factor a quadratic equation in a number of ways:

- Use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Use FOIL
- Complete the square
- Use grouping

$ax^2 + bx + c = 0$

$2x^2 - 6x^2 - 56 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 2, b = -6, c = -56$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(2)(-56)}}{2(2)}$$

$$x = \frac{6 \pm 22}{4}$$

$x = 7, -4$

$x^2 - 3x - 28 = 0$

What are the factors of -28 that add up to -3?

28

1 28

2 14

4 7

$(x - 7)(x + 4) = 0$

$x - 7 = 0;\ x = 7$

$x + 4 = 0;\ x = -4$

4 and -7 are the pair that we want